



# SUPERCONDUCTING 2-D METAMATERIALS: ANALOGY TO STACKED TUNNEL JOSEPHSON JUNCTIONS

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**Abstract** – We present concept and results of numerical development of active superconducting metamaterials, which can generate and amplify the high-frequency signals. We suggest that equivalent schemes of our experimental underdamped and overdamped 2-D circuits with Josephson junctions can be applied for analysis of naturally stacked Josephson junctions (layered superconductors crystals). Both the THz-range oscillator and the 10-GHz range SQUID-amplifier are limited by their upper Josephson frequency and by compactness of their 4-junction cell. Methods of reducing the size of the cell including advanced design of shunt resistors are described and discussed along with detailed electromagnetic modeling and analysis.

## I. INTRODUCTION

The use of stacked tunnel Josephson junction (STJJ) based on natural crystals of superconductors is of great interest for terahertz range oscillators Ref. [1]. These 3-D oscillators operate at geometry-defined modes of the dielectric cavity of the crystal. The series connected junctions are known to suffer from unequal Josephson frequencies, unless they are synchronized via external RF network Ref. [2]. From the first sight, the STJJ are naturally equal, so they do not need any external network. However, the synchronous regime of STJJ occurs along with heating up the bias region above  $T_c$ , that provide an internal resistive circuit shunting all layers, as illustrated in Fig. 1. These support the old rule of need of a circuit, which can be a common load combining all JJ via either series current or parallel voltage or via combination of current and voltage. We are going to establish and develop an approach to STJJ via planar (2-D) metamaterial.

## II. 2-D ANALOGY TO A STACKED JUNCTION

The active metamaterials based on planar 2-D JJ arrays can offer, in principle, unlimited growth of both power and saturation levels. According to schemes from Fig. 1, the 2-D infinitely expandable arrays look very similar to STJJ. The 2-D "replacement" for oscillators have been demonstrated long time ago at about 170 GHz Ref. [3, 4]. It was found that the frequency of the oscillator is rather fixed at the internal resonant mode that is quite similar to STJJ. However, the internal resonance mode of the 2-D array is defined with rather different parameters of its geometry. The initial resonant 2-D cell contains 4 JJ interacting with superconducting ground plane as demonstrated in Ref. [5]. One may call such circuit "a reactive shunt" as illustrated in Fig. 2a. Notice here that the geometry of such shunt is lithography defined, so it can be very accurate. We have estimated that a 2-D array-oscillator similar to one from Refs. [3, 4] can perform at higher frequencies up to 1 THz Ref. [5].

## III. ROLE OF SHUNT RESISTOR FOR A JOSEPHSON EFFECT AMPLIFIER

The combination of high amplitude and high frequency of Josephson current is known to be the important issue for a SQUID-amplifier, see Refs. [5, 6]. A 3-D SQUID-amplifier might be suggested, if one could shunt the STJJ layers. Obviously, it is not possible in a direct way. However, according to our metamaterial approach, the replacement of a superconducting ground plane with a resistive sheet beneath the structure, as illustrated in Fig. 2b, makes sense. It is demonstrated in Ref. [5], that 2-D expansion may improve the operation of a dc-

SQUID sensor. The basic idea is to provide a common shunt resistor for all JJ of the 2-D SQUID structure as shown in Fig. 2b. Such shunt facilitates the synchronization of all JJ, while providing minimum possible inductance of the shunting circuit. Keeping the design inductance of a shunt at its minimum is helpful for achieving the maximum possible value of the critical voltage,  $V_C$ , which is responsible for highest possible bias voltage of JJ, if such shunt is connected to a real circuit. The high value of  $V_C$  is resulting also in smooth IV-curve and higher gain of a SQUID-amplifier. One may compare the new approach with the traditional way of shunting JJ as required by most design rules, see for example Ref. [7] using the following relation

$$\beta_c = \frac{2e}{h} I_C C R_N^2 \quad (1)$$

Here  $I_C$  is critical current of JJ,  $C$  – capacitance of JJ,  $R_N$  – effective junction resistance in the normal state including shunt resistance.

The comparison of effective  $V_C$ , derived from Fig. 3, is demonstrating the advantage of the new approach in shunting the 2-D JJ array neither on top nor individually, but with the common resistive layer beneath the structure.

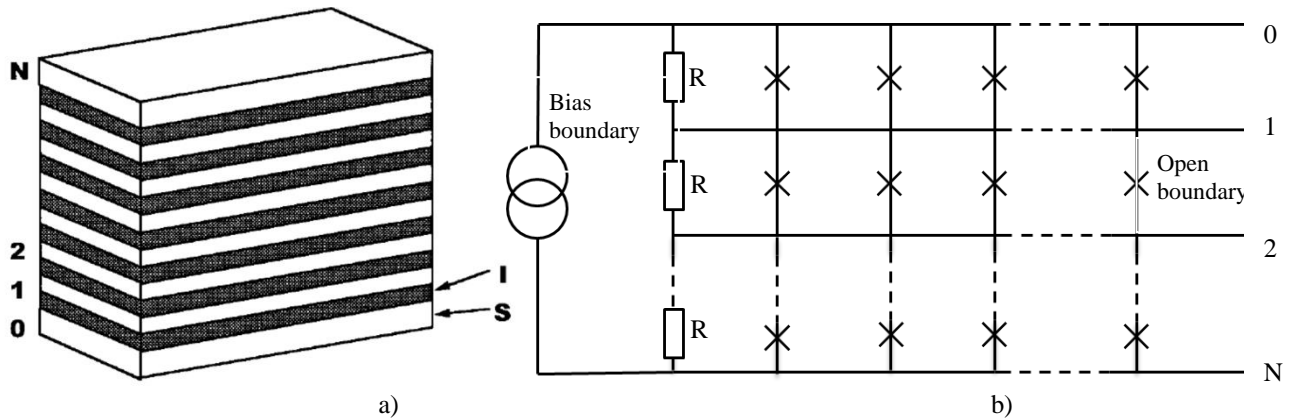


Fig. 1. Stacked Josephson junction. a) Simplified layout of stacked Josephson junction (3-D view), where "I" represent insulator layer, S – superconductor layer, N – number of layer. b) Equivalent electrical schematic. Each row represents a tunnel layer, each column - discrete element of a tunnel layer ("X" represents a Josephson junction, R is resistivity of overheated JJ).

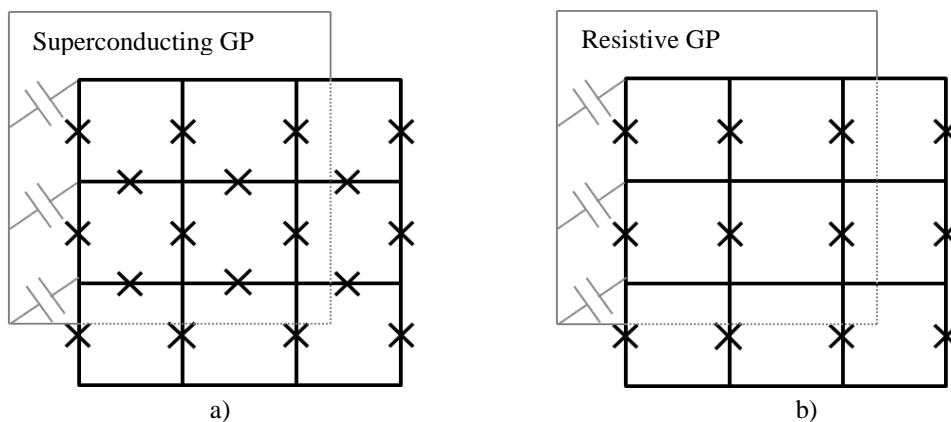


Fig. 2. Variants of 2-D Josephson junction array. a) Oscillator array with superconducting ground plane. b) Amplifying array with resistive ground plane. Capacitors represent reactive currents existing at all nodes of the structure due to presence of the ground plane (not all node capacitors are shown).

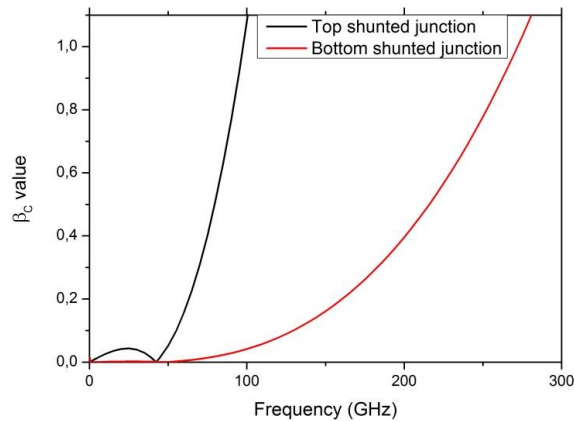


Fig. 3. McCumber parameter (1) calculated using EM-model for two variant of resistive shunt. It can be seen that the shunt made as the bottom layer offers wider frequency range.

#### IV. CONCLUSION

The analogy between stacked tunnel Josephson junctions and 2-D metamaterial arrays of discrete Josephson junctions is argued. The new approach can explain some important features and relation between many Josephson structures yet demonstrating new hints in designing superconducting devices for both generation of terahertz-range power and amplification of weak gigahertz signals.

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