Quantum Left-handed metamaterial with superconducting quantum interference devices

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A scheme of left-handed metamaterial (LHM) composed of superconducting quantum interference devices (SQUIDs) and conducting wires is proposed. The permeability of a probe field can be smoothly tuned over a wide range with another electromagnetic (coupling) field due to quantum interference effect. Similar to electromagnetically induced transparency (EIT) of atomic systems, the absorption of the probe field can be strongly suppressed even in the case of negative permeability. There are two passbands of negative refractive index with low loss, which can be tuned with the coupling field.

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Recently, research on left-handed materials (LHMs)[1] has attracted considerable attentions. The LHMs are a kind of metameterial which have negative permittivity and negative permeability , which lead to negative refractive index in a narrow frequency band. Many novel phenomena can occur on LHM such as superprism, perfect flat lens, inverse light pressure, and reverse Doppler and Vavilov-Cherenkov effects . Large nonlinearity can also occur, e.g. the bistable transition of permeability from positive to negative[2]. The quantum phenomena in LHMs have also attracted attentions such as the modified spontaneous emission of atoms in LHM [3, 4]. However, the background of the ordinary LHM such as array of conductor lines and slit-ring resonators (SRRs)

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is a 'classical' system where the negative ϵ and negative μ arise from the classical plasma oscillations. In this sense, the ordinary LHMs are quite different from quantum systems such as atomic gases. On the other hand, in ordinary situations, the magnetic response of atomic gases to a laser field is too weak to generate negative permeability. How to enhance the magnetic response of a quantum system is essential to realize LHMs at optical frequency band. There may be other ways to obtain negative permeability, e.g., recently, a scheme of electromagnetically-induced left-handedness in atomic gases is proposed[5], where large density of atoms is necessary and special restrictions on frequencies of driven fields and atomic transitions are required. In contrast, an artificial quantum system with high magnetic sensibility can has much more advantage than atomic systems to realize negative permeability.

In this letter we propose a new kind of LHM composed of superconducting rings with Josephson junctions and conductor wires. For simplicity, the wires are assumed to be normal conductors. It should be note that quite different from our work, recently analysis and design of superconducting transmission lines are presented by Salehi et al [6], and more recently, Ricci et al have experimentally researched on a metamaterial that employs superconducting Nb metals and low-loss dielectric materials, in which case negative effective index passband are seen between 50MHz to 18GHz [7]. The focus of our work is on the negative permeability which arises from quantum feature of the composite. In contrast with the 'classical' SRR, in our model the split of the ring is replaced by a Josephson junction, which is essential to the quantum feature of the LHM. We will show analytically that due to the quantum interference effects[8] the permeability can be tuned over a wide range with an external microwave field. Also, we will show that the absorption of the medium for the probe field can be strongly suppressed even in the case of negative refractive index.

A scheme for supperconducting LHM is show in Fig.1, where the composite is composed of superconducting rings with Josephson junctions, and, an effective electric medium with effective electric permittivity ϵ as a background, which, e.g., can be an array of normal conducting wires. The radius of the ring is denoted by a, and the period of the array is denotd by d. Schematic of the potential energy and the first six eigen energies of the SQUID is shown in Fig. 2. We assume a probe microwave field is interacting with the composite. The wavelength of the field is assumed to be much longer than the period, i.e., $d << \lambda$. For a classical cylindrical SRR system, the permeability μ can be given by the relation (according to Ref.[2])

$$\mathbf{B}(\omega) = \mathbf{H}_x(\omega) + FH'(\omega), \tag{1}$$

where $H_x(\omega)$ is the alternating external magnetic field and $H'(\omega)$ is the additional magnetic field induced by $H_x(\omega)$, which determines the magnetization of the composite, and $F = \pi a^2/d^2$ is the fraction of the structure with a being the radius of the ring and d being the periodicity of the array. Also, $a << d << \lambda$ is assumed.

The permeability at angular frequency ω can be given by

$$\mu(\omega) = 1 + F\frac{\phi(\omega)}{\phi_x(\omega)}, \tag{2}$$

where $\phi(\omega)$ is the flux of frequency ω induced by the external microwave field, i.e. $\phi(\omega) = H's$ with $s = \pi a^2$ being the area of the ring. For the quantum LHM here, the SRRs are replaced by the SQUIDs, then $\phi(\omega)$ becomes an operator although the external driven fields are assumed to be classical. In order to calculate μ, $\phi(\omega)$ in Eq. (2) should be replaced by the quantum averaging of it, i.e. $< \phi(\omega) >$.

The Hamiltonian of a SQUID (a ring with a Josephson junction) can be given by[9]

$$H_0 = -\frac{\hbar}{2m}\frac{\partial}{\partial x^2} + V(x) \tag{3}$$

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with the potential of the SQUID being

$$V(x) = \frac{1}{2}m\omega_{LC}^2(x - x') - \frac{1}{4\pi^2}m\omega_{LC}^2\beta cos(2\pi x) \tag{4}$$

where $x = \phi/\phi_0$, $m = C\phi_0^2$, $\omega_{LC}^2 = \frac{1}{LC}$, $\beta = 2\pi LI_c/\phi_0$, and $x' = \phi_x/\phi_0$. Here ϕ is the total flux in the ring, L is the ring inductance, ϕ_x is an external applied magnetic flux to the SQUID, I_c is the critical current of the junction, C is the capacitance of the junction, and $\phi_0(= h/2e)$ is the flux quantum. We consider a realistic SQUID system which can be described by use of the parameters as in the work of Zhou etal [9], where $L = 100pH$, $C = 40fF$, and $I_c = 3.95\mu A$, leading to $\omega_{LC} = 5 \times 10^{11} rad/s$, and $\beta = 1.2$. The external DC magnetic field parameter x' is taken to be -0.501.

The interaction between the SQUID and microwave fields, which are assumed to be linearly polarized with their magnetic field perpendicular to the plane of the SQUID ring, is described by the time dependent potential

$$V_{int}(x,t) = m\omega_{LC}^2(x - x')(\varepsilon cos\omega t + \varepsilon_c cos\omega_c t). \tag{5}$$

In the interaction picture ,the dynamics of the system is governed by the Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}|\psi(t) >= V_{int}|\psi(t) > . \tag{6}$$

Here ε, ε_c are microwave magnetic flux of the probe field and the coupling field in units of ϕ_0. The frequency of the probe field and the coupling field are chosen near resonant with transition $|0 > -|n >$ and $|1 > -|n >)$ respectively, where $|n >$ denotes the n-th eigen state of H_0. This is usually referred to as the three-level approximation. In this case, the wave function can be written as

$$|\psi(t) >= c_0(t)|0 > +c_1(t)|1 > +c_n(t)|n >, \tag{7}$$

and the interaction Hamiltonian after the rotating-wave approximation, can

be written as [9]

$$V_{int} = \hbar(\Omega e^{i\delta t}|0><n|+\Omega_c e^{-i\delta_c t})|1><n|+h.c.-i\frac{\gamma_1}{2}|1><1|-i\frac{\gamma_n}{2}|2><2|). \tag{8}$$

Here the Rabi frequency Ω and Ω_c are defined as

$$\Omega = -x_{0n}m\omega_{LC}^2\epsilon/\hbar, \quad \Omega_c = -x_{1n}m\omega_{LC}^2\epsilon_c/\hbar, \tag{9}$$

where x_{0n} and x_{1n} are defined as $<0|x|n>$ and $<1|x|n>$ respectively. Also, here $\delta = \omega - \omega_{0n}$, $\delta_c = \omega_c - \omega_{1n}$, γ_1 and γ_n are the background decay rate of the state $|1>$ and $|n>$. The equations for amplitudes c_0, c_1, and c_n can be easily obtained from Eq.(6) as

$$\begin{aligned}
i\frac{dc_0}{dt} &= \frac{\Omega}{2}c_n \\
i\frac{dc_n}{dt} &= -(\delta + i\frac{\gamma_n}{2})c_n + \frac{\Omega}{2}c_0 + \frac{\Omega_c}{2}c_1 \ , \\
i\frac{dc_1}{dt} &= -(\delta - \delta_c + i\frac{\gamma_1}{2})c_1 + \frac{\Omega_c}{2}c_n
\end{aligned} \tag{10}$$

We assume that $\gamma_1 << \gamma_n$, where γ_n can be taken to be about 1GHz according to Ref. [10, 8]. We assume that initially the system is prepared in the ground state $|0>$, and for simplicity, we assume the probe field is weak enough, i.e. $\Omega << \gamma_n, \Omega << \Omega_c$. In this case the steady-state solution can be obtained as in ordinary EIT systems.

It is easy to prove that the mean value of the flux $(<\phi(\omega)>)$ can be given by

$$\frac{<\phi(\omega)>}{\phi_x(\omega)} = -\alpha\frac{c_0^*c_1}{\Omega}, \tag{11}$$

where $\alpha = \frac{m\omega_{LC}^2 x_{0n}^2}{\hbar} = \frac{\phi_0^2}{\hbar L}x_{0n}^2$. Here α is a parameter measures the sensibility of the effective medium to the probe field, which has the dimensions of frequency (Hz). For $n = 4$, $x_{04} = 5.39798 \times 10^{-3}$, which is given by Ref.[11, 12]. Taking $L = 100pH$, we obtain $\alpha = 11.815$GHz. This value will be used in the following analysis.

We first consider the case of $\Omega_c = 0$, i.e. of two-level system (non-EIT

system). According to Eq.(2),(11) and (10), the permeability can be give by

$$\mu = 1 - \frac{F}{2} \frac{\alpha}{\delta + i\gamma_4/2} \tag{12}$$

, It is clearly a Lorentz profile. When the probe field is near resonant with the transition $|0> - |n>$, strong absorption will occur. The absorption becomes sharp with the decrease of γ_4. Similar to classical SRR composite, negative μ can occur (see Fig. 3(a)) if α is large enough. The condition for the negative μ can be easily deduced from Eq. 12 as that $F\alpha > 2\gamma_4$. Also, it is clear that there is only one frequency band of negative $\text{Re}(\mu)$. However , it is worth to note that the μ-spectrum for the quantum composite here is similar but not the same as that in classical composite[2].

If the coupling field is applied to the system ($\Omega_c \neq 0$), the magnetic response could be significantly modified. For a strong coupling field, i.e. $\Omega_c >> \Omega$, γ_4, from Eq.(2),(11) and (10), the permeability can be given by

$$\mu = 1 - \frac{F}{2} \frac{\alpha \left(\delta - \delta_c + i\frac{\gamma_1}{2}\right)}{\left(\delta - \delta_c + i\frac{\gamma_1}{2}\right)\left(\delta + i\frac{\gamma_4}{2}\right) - \frac{\Omega_c^2}{4}}. \tag{13}$$

The spectrum of μ in three-level case is show in Fig. 3(b). It can be easily seen that the transparency occurs when $\delta = \delta_c = 0$, which is similar to EIT of atomic system except that here the coupling field is used to control the permeability μ instead of the permittivity ϵ of the probe field. Superconductive analog to electromagnetically induced transparency that utilizes superconductive quantum circuit designs of present day experimental consideration has also been investigated by Muralial et al [8]. The width of the transparency window can be defined by the frequency distance between the two absorption peaks which is about Ω_c due to Autler-Townes splitting. The frequency band for low absorption and large refraction can occur over a wide range for large Ω_c. On the contrary, the group velocity of the probe field can be reduced to near zero as in atomic EIT systems. If $F\alpha$ is very small, μ is positive for all probe detunings (δ), which can be easily seen from Eq.(13). If $F\alpha$ is large

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enough, however, negative $\text{Re}(\mu)$ can occur near the transparency window. The condition for negative μ can be deduced from Eq. (13) as that

$$F\alpha > 2\gamma_4. \tag{14}$$

This condition is the same as that for nonEIT case, but the frequency band for negative μ in the EIT case and nonEIT case is quite different. In the EIT case there are two frequency bands for negative μ, which can be given by

$$f_-(x_-) < \delta/\gamma_4 < f_-(x_+); \quad f_+(x_-) < \delta/\gamma_4 < f_+(x_+), \tag{15}$$

where

$$f_\pm(x) = \frac{x \pm \sqrt{x^2 + \Omega_c^2/\gamma_4^2}}{2}, \tag{16}$$

and

$$x_\pm = \frac{1}{2}(g \pm \sqrt{g^2 - 1}) \tag{17}$$

with $g = \frac{F\alpha}{2\gamma_4}(> 1)$

For example, we consider the case of $\alpha = 11.815\text{GHz}$ as above. If taking $F = \pi \times 0.14$, we can obtain that $F\alpha/2 = 2.5982\text{GHz}$. In this case negative $\text{Re}(\mu)$ can occur if the condition $\gamma_4 < 2.5982\text{GHz}$ is met.

It should be note that, when $\delta = \delta_c$ and $\gamma_1 = 0$, exactly zero absorption occcur ($\text{Im}(\mu) = 0$), but in this case $\text{Re}(\mu) = +1$. When $\text{Re}(\mu)$ is negative, $\text{Im}(\mu)$ is always nonzero, i.e. negative refractive index without loss can never occur. However, the absorption can be strongly suppressed near the condition of $\delta = \delta_c$. It is similar to ordinary EIT in atomic systems, but here the zero absorption occurs due to magnetic response instead of electric response, and, here, the magnetic is so strong that negative $\text{Re}(\mu)$ can occur near EIT condition. The permeability (μ) can be smoothly tuned over a wide range with the coupling detuning δ_c (see Fig. 4) and its Rabi frequency Ω_c (see Fig.(5)). It is worth to note that the tunability is due to the quantum interference effect which is absent in classical normal conductor systems. The permeability μ

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and refractive index n can be smoothly tuned by the coupling field over a wide range, and the continuous transition from $\text{Re}(\mu) = 1$ to $\text{Re}(\mu) = -1$ is possible. From the μ-spectrum it is easy to see that there are four frequencies for $\text{Re}(\mu) = -1$, but only two of them are for low loss (small $\text{Im}(\mu)$), one of which is in EIT window, while the other one is out of EIT window. It is should be note that both of them can be tuned with the coulpling field. For this two frequencies, $\text{Im}(\mu) \to 0$ when $\gamma_4 \to 0$ and $\gamma_1 \to 0$.

In order to calculate the refractive index, the permittivity ϵ and permeability μ can be written as

$$\epsilon = |\epsilon|e^{i\phi_e}, \mu = |\mu|e^{i\phi_m}, \tag{18}$$

where $0 < \phi_e < \pi$, $0 < \phi_m < \pi$. Then the refractive index n can be given by [3]

$$n = \sqrt{|\epsilon||\mu|}e^{i(\phi_e + \phi_m)/2} \tag{19}$$

For the sake of simplicity, we consider the case where the effective electric permittivity ϵ is generated by an array of normal conducting classical current wires[3, 2]. In this case

$$\epsilon = 1 + \frac{\omega_{pe}^2}{\omega_{Te}^2 - \omega^2 - i\omega\gamma_e} \tag{20}$$

where ω_{pe} and ω_{Te} are the parameters which measure the effective plasma oscillations and γ_e measures the loss of the wires. The refractive index spectrum of the composite is shown in Fig. (6), where (a) is for $\epsilon(\delta)$, (b) is for $\mu(\delta)$, and (c) is for $n(\delta)$. For simplicity, we consider the frequency band of negative ϵ which is far from resonance, in which case ϵ is slowly varing with δ, and $\text{Im}(\epsilon)$ is small. We find in this case there are two minima on the $\text{Im}(n)$-spectrum. The absorption minima arise from two reasons: (1) small $\text{Im}(\epsilon)$ and small $\text{Im}(\mu)$; (2)$\text{Re}(\mu)$ and $\text{Re}(\epsilon)$ are simultaneously negative. On the contrary, if $\text{Re}(\mu)Re(\epsilon) < 0$, $\text{Im}(n)$ will be large even if both of $\text{Im}(\mu)$ and $\text{Im}(\epsilon)$ are in-

finitely small. Two passbands of negative refractive index appear around the two minima, both of them are sensitive to the coupling field.

In conclusion, we have analytically investigated the metamaterial composed of superconducting rings with Josephson junctions and conducting wires. It is found that negative permeability for a probe microwave field can occur when $g = \frac{F\alpha}{2\gamma_4} > 1$. There are two passbands of negative refractive index with low loss, which can be tuned with the coupling field. To our knowledge, it is for the first time that the quantum left-handed metamaterial being composed of SQUIDs has been investigated. One of the advantages of this kind of LHM is that the negative refractive index with low loss is easy to obtain due to that the permeability can be smoothly tuned over a wide range including $\mu = \pm 1 + i\varepsilon$ ($\varepsilon \to 0+$). In the quantum composite, some new physics could be found, such as that associated with transient properties of the left-handedness, large nonlinearity due to the quantum interference effect and that due to strong magnetic response of the composite. Also, the tunability of the composite via quantum interference effect could be used in quantum information process [13]. On the other hand, the LHM with SQUIDs can be regarded as a bridge between the 'classical' LHMs (e.g. composed of SRRs and wires) and the 'quantum' systems composed of microscopic particles(e.g. atoms or quantum dots). Hence, further research on the quantum LHM could facilitate the realizing of the tunable LHMs of high-frequency (towarding to optical) band.

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Figure Captions

Fig. 1. Schematic of the composite metamaterial structure composed of superconducting rings with Josephson junctions (SQUIDs) and the effect electric medium such as conductor wires. Each SQUID is an artificial Λ-configuration three-level system which coupled with a probe microwave field and a coupling microwave field.

Fig. 2. Schematic of the potential energy and the first six eigen energies of the SQUID. The energies of the ground state $|0>$, meta-stable state $|1>$, and the excited state $|4>$ are 7.81984mev, 7.90183mev, and 8.14057mev, respectively.

Fig. 3. Real part (solid curve) and imaginary part (dashed curve) of the permeability (μ) versus the probe detuning δ for $\delta_c = 0$, where $F = \pi \times 0.14$, $\gamma_4 = 0.0423$, $\gamma_1 = 0.1\gamma_4$, $\beta = 1.2$, $x' = -0.501$. (a) is for $\Omega_c = 0$ and (b) is for $\Omega_c = 0.5078$. All parameters are in units of α, which is taken to be 11.815GHz.

Fig. 4. Real part (solid curve) and imaginary part (dashed curve) of the permeability (μ) versus the coupling detuning δ_c for the case of $\delta = 0$. The parameters are the same as that in Fig. 3

Fig.5. The band edges of negative $\mathrm{Re}(\mu)$ versus the Rabi frequency of the coupling field Ω_c. Negative $\mathrm{Re}(\mu)$ occurs when δ is between the two solid curves (band1) or that between the dashed curves(band2). The parameters are the same as that in Fig.3.

Fig.6. The spectrum of permittivity(a), permeability(b), and refractive index(c), where $\omega_{pe} = 1.5\omega_{04}$, $\omega_{Te} = 0.43\omega_{04}$, $\gamma_e = 0.1\omega_{04}$. The solid line is for the real part of them, while the dashed line is for the imaginary part of them. Here $\omega_{04} = 48.727$GHz. Other parameters are the same as that in
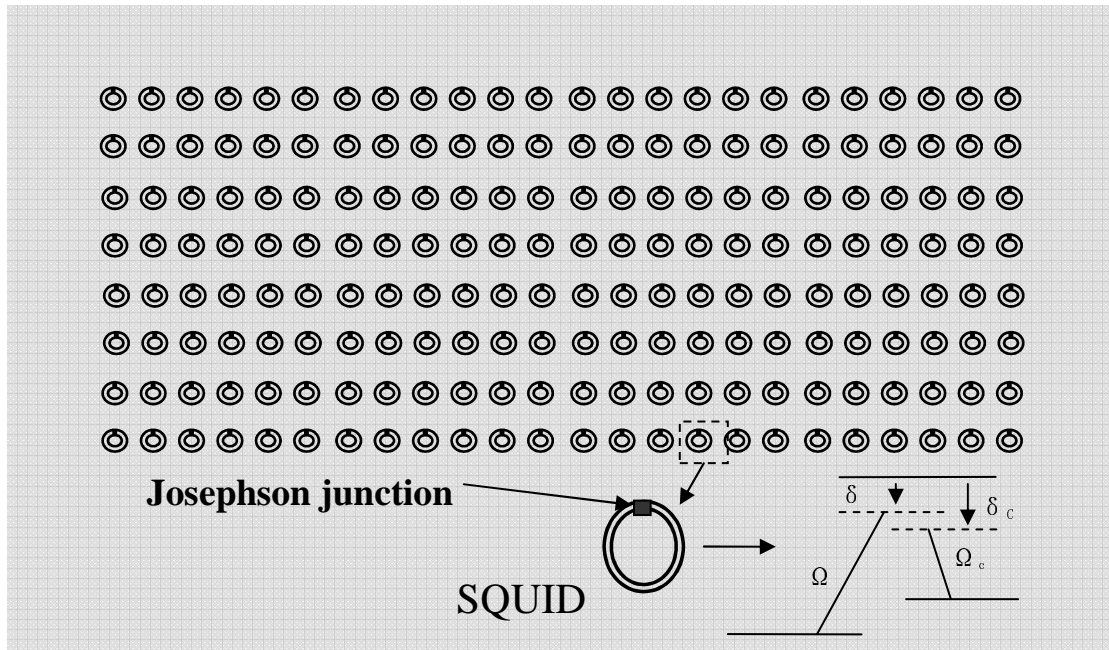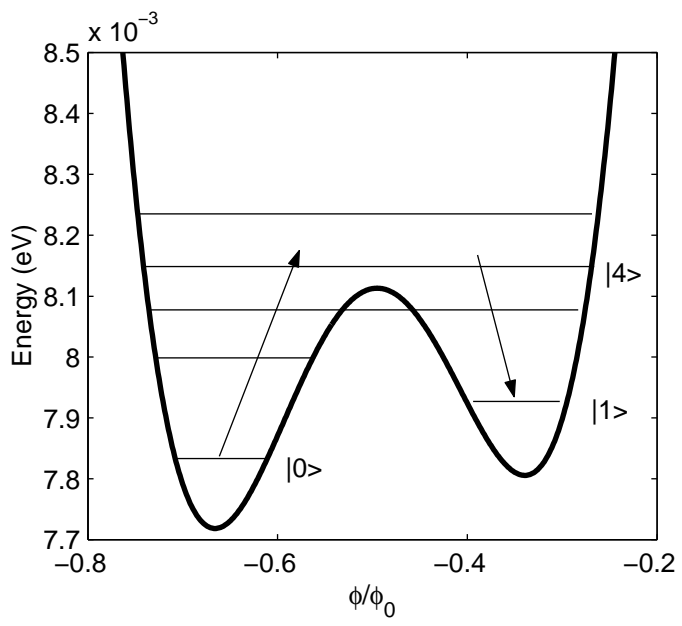
Fig.3.

Josephson junction

SQUID

δ δ_c

Ω Ω_c

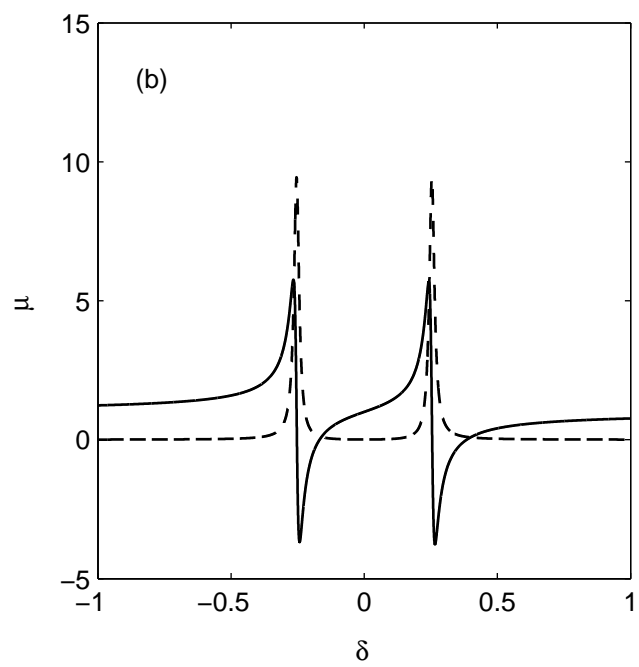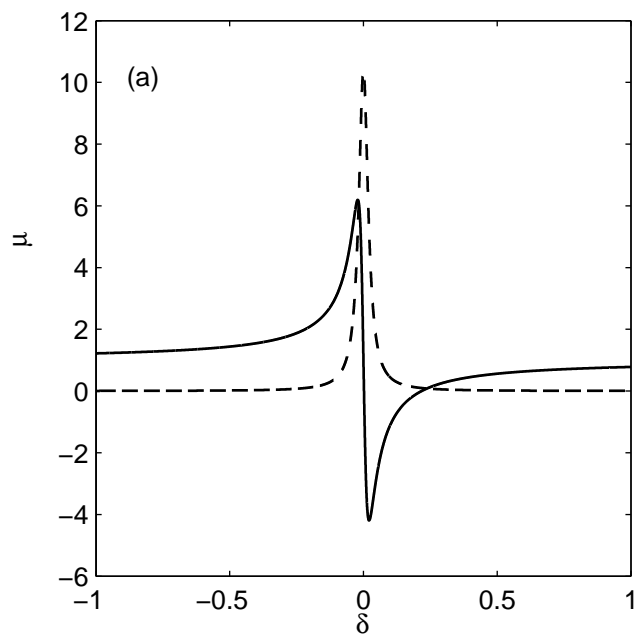Fig. 1   Du *et al*

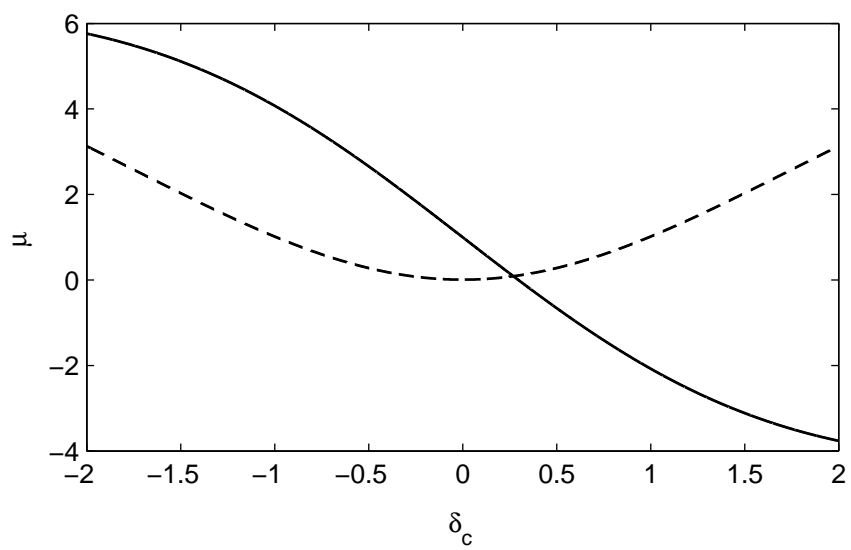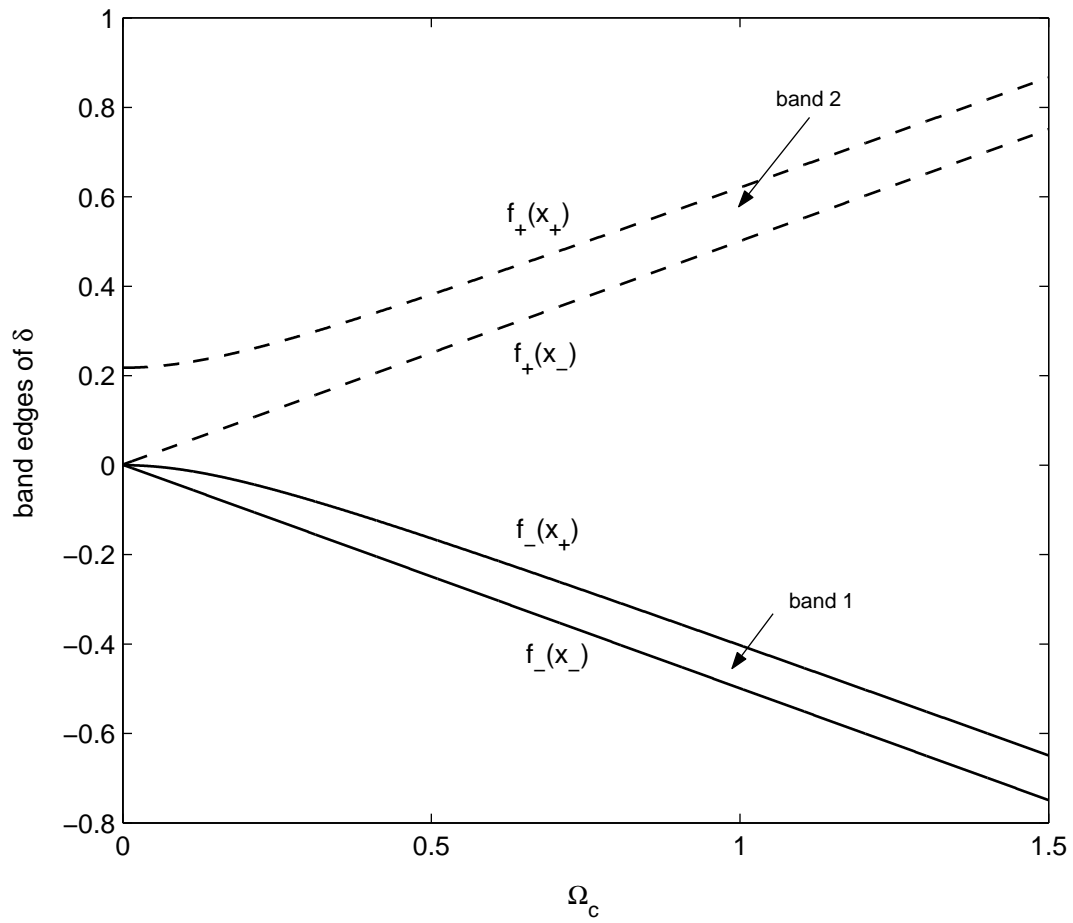Fig. 2   Du et al.

Fig.3

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Fig. 4

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Fig. 5   Du et al.

Fig. 6

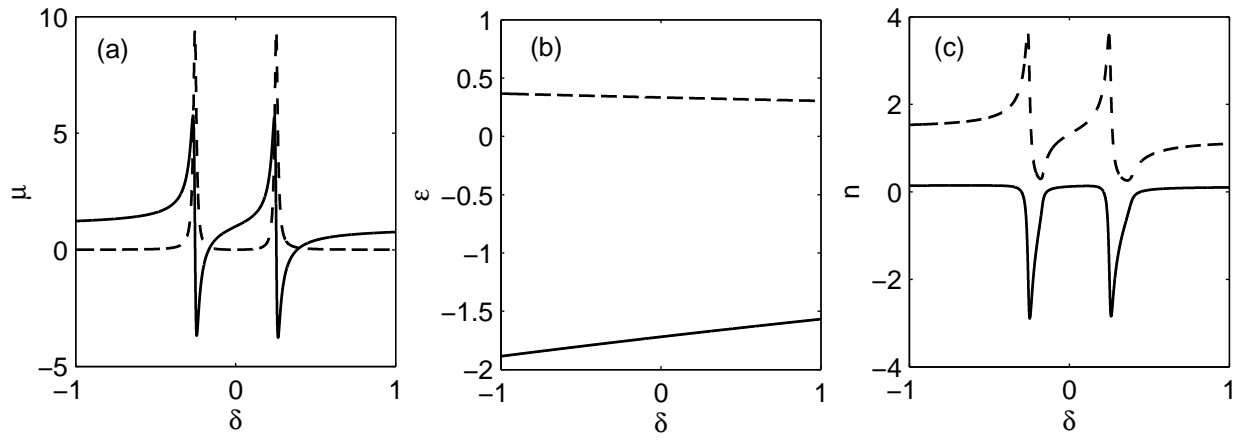Du et al.