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Electrodynamics of a spiral resonator as a suitable magnetic component of metamaterials

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Abstract – **We consider spiral resonators as promising ultra-compact magnetic components for negative index metamaterials. In order to explore such a possibility, we study the electrodynamics of a planar monofilar Archimedean superconducting spiral resonator. The frequencies of microwave resonances that can be excited in such a system are calculated taking into account an inhomogeneous current distribution across the spiral. It is shown that the resonance frequencies are equidistant, and the current distribution inside of the spiral can be presented in a simple analytical form. The obtained values of resonance frequencies are in a good agreement with experiment.**

I. Introduction

Metamaterials are associated with artificially prepared and structured media showing unusual electrodynamic properties, e.g. left-handed wave propagation, negative index of refraction, variety of photonic patterns, etc. [1]. The design of such materials established over decade ago consists of a set of resonators incorporating a transmission line [2, 3]. In the many previous experiments, various types of split-ring resonators (SRRs) have been used [3, 4, 5]. The resonant frequencies of such resonators are determined by the ratio between the width of the capacitor gap ℓ and the size R of SRR, and are typically in the GHz range.

The greatest advantage of a multi-turn spiral resonator is its compactness. The characteristic dimensions of a spiral resonator could be orders of magnitude smaller then its resonant wavelength. The major limitation to compactness of resonators made of normal metals are the losses, which increase with decreasing the resonator size. Planar spiral resonators (PSRs) made of superconducting materials easily overcome this problem and can be made deep sub-wavelength in size [5, 6]. Recent experiments with superconducting PSRs showed high-Q resonances in 100 MHz range [6, 7]. Using monofilar PSRs with densely packed turns, the lowest resonances were observed at the frequencies about 70 MHz [7]. For a ring-shaped *narrow spiral*, with both inner R_i and outer R_e radii larger than the width of the spiral, a peculiar dependence of resonant frequencies on the resonance number n, that resembles $\omega_R^n \propto (n + 1/2)$, has been experimentally observed. Here, we consider another limit of a disk-shaped *wide spiral* resonator having the radii $R_i \ll R_e$.

As we turn to the theoretical analysis of electrodynamic properties of a PSR, it is necessary to find resonant frequencies and current distribution inside of the PSR. For superconducting PSRs, the ac current distribution across a spiral can be experimentally measured by the laser scanning microscopy. These measurements carried out in Ref. [7] have shown strongly inhomogeneous current distribution as the system was tuned to the resonance. Therefore, to quantitatively analyze resonant frequencies with an arbitrary harmonic n, one has to elaborate a theoretical analysis of spiral resonator electrodynamics taking into account inhomogeneous current distribution across a spiral. The time-dependent vector-potential $\vec{\mathbf{A}}(\vec{\mathbf{r}}, z, t)$ and electric field $\vec{\mathbf{E}}(\vec{\mathbf{r}}, z, t)$ are determined by the electrical current flowing along a spiral, $\vec{\mathbf{j}}(\vec{\rho}, t)$, where $\vec{\mathbf{r}}$ and the coordinate z determine the position of the observation point. The vector $\vec{\rho}$ determines the coordinate in the plane of spiral. Using specific boundary conditions, i.e. the component of electric field parallel to the spiral wire $\vec{\mathbf{E}}_{\|}$ to be zero, we can obtain a set of resonant frequencies.

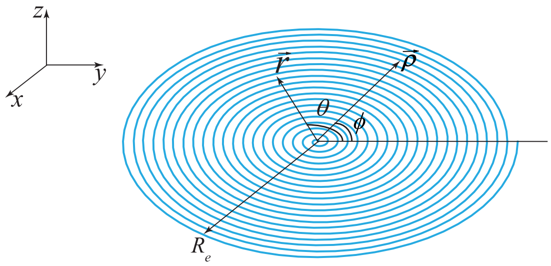

Fig. 1: A sketch of a spiral resonator with many turns. The polar coordinates ρ, φ (the coordinates of the point on spiral), and r, θ (the coordinates of the observation point in the plane of the spiral) are shown. R_e is the external radius of spiral.

II. Electrodynamics of a thin wide planar spiral resonator

Let us consider a monofilar Archimedean spiral resonator of a finite length with many densely packed turns. The total number of turns is N. The current $\vec{\mathbf{j}}(\vec{\rho}, t)$ flows along the spiral. Such geometry can be characterized by the angle φ which changes from 0 to $2\pi N$, and a corresponding change of a polar coordinate ρ. The equation of Archimedean spiral is written as

$$\rho(\varphi) = R_e(1 - \alpha\varphi), \tag{1}$$

where the parameter $\alpha = d/(2\pi R_e) = (R_e - R_i)/(2\pi R_e N) \ll 1$ and R_e and R_i are accordingly the external and internal radii of a spiral, and d is the distance between adjacent turns. Here we consider a *wide* PSR with $R_i \ll R_e$. The schematic view of the spiral resonator is shown in Fig. 1. The following electrodynamic analysis of a thin wide planar spiral resonator is similar to the one carried out in Refs. [8, 9] for an infinite helical coil. We neglect the current inhomogeneities inside of the wire and use the following approximation: The wave vector k is much smaller than a typical inverse size of inhomogeneities in the current distribution across a spiral $\psi(\rho)$, i.e. $k \ll 1/R_e$.

In order to obtain resonant frequencies we apply boundary conditions specific to the spiral case, i.e. the component of electric field parallel to the wire E_s is equal zero. This condition can be written as

$$R_e \alpha E_r + r E_\theta|_{z=0} = 0, \tag{2}$$

where E_r and E_θ are an electric field components. Both the electric field and the vector-potential are determined by current distribution function $\psi(\rho)$. Thus, Eq. (2) could be rewritten to following differential-integral equation for new function $Y(\tau) = e^{-\tau}\psi(\tau)$:

$$-Y'_\tau(0)K(\xi) + \int_0^\infty d\tau K(\xi - \tau)\Big(Y''_\tau(\tau) + 2Y'_\tau(\tau) + \frac{\omega^2\epsilon_0\mu_0 R_e^2}{\alpha^2} e^{-3\xi} Y(\tau) e^{-\tau}\Big) = 0, \tag{3}$$

where $\tau = \ln\frac{R_e}{\rho}$, $\xi = \ln\frac{R_e}{r}$ and the kernel $K(\xi - \tau) = \int_0^\infty dz J_1[e^{\xi-\tau}z]J_1[z]$. Within local approximation, Eq. (3) is written as

$$Y''(\xi) + 2Y'(\xi) + \frac{\omega^2\epsilon_0\mu_0 R_e^2}{\alpha^2} e^{-4\xi} Y(\xi) = Y'(0)K(\xi). \tag{4}$$

The solution of homogenies equation (4) is $Y(\xi) = A\sin(\frac{\omega}{2}\frac{R_e}{c\alpha}e^{-2\xi})$ which means that the current distribution is written as

$$\psi(r) = A\frac{R_e}{r}\sin(\frac{\omega}{2}\frac{R_e}{c\alpha}r^2), \tag{5}$$

where A is an integration constant. There are specific boundary conditions $\psi(0) = \psi(1) = 0$, which allow one to find the resonant frequencies. The resonant frequencies are determined by the formula $\omega_n = 2\pi n\frac{\alpha}{cR_e}$. The graphic representation of Eq. (5) at resonant frequencies is shown in Fig. 2.

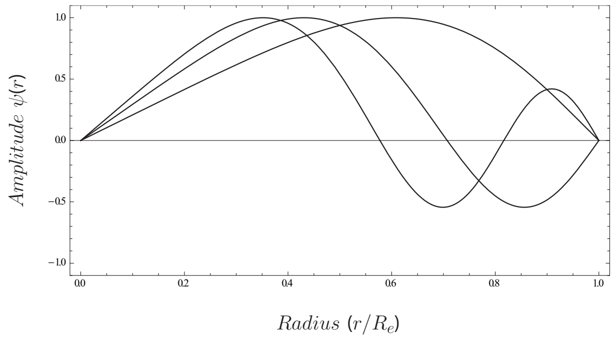

Fig. 2: The current distribution across the spiral turns for first three resonance modes. $\psi(r)$ normalized to maximum value equal to one.

III. Experimental results

We have measured the transmission spectrum of a single superconducting spiral fabricated as a printed circuit board covered by thin layer of Pb-Sn alloy with dimensions $R_e = 15$ mm, $N = 23$, $d = 0.7$ mm. The sample is placed between two loop probes $30mm$ in diameter, whose axes are parallel to each other and the spiral, as it was made in recent work [6]. The distance between the loops and the spiral is $40mm$. The data are taken at temperature $1.6K$ with an input power of 10 dBm and demonstrate five resonances at 84 MHz, 184 MHz, 273 MHz, 360 MHz and 446 MHz. These results are in agreement with theoretically predicted resonant frequency relation $1:2:3:4:...$ except of the first harmonic frequency which is lower than expected. Lowering of the first resonant frequency could be qualitatively explained by the coupling between the spiral and the probe loops.

IV. Conclusion

Theoretical model and experimental results for a single Archimedean superconducting spiral resonator are reported. The analytically calculated resonant frequencies are found to be in good accord with the experiment. The profile of the current distribution across a spiral at resonance frequencies is also calculated. The current distribution shows a pronounced maximum shifted to the outer side of the spiral. The medium of spiral resonators could be used as a magnetic component of negative index metamaterials.

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References

[1] N. Engheta, R. W. Ziolkowski, *Metamaterials: Physics and Engineering Explorations* , Wiley & Sons, New York, (2006); S. Zouhdi, Ari Sihvola, Al. P. Vinogradov, *Metamaterials and Plasmonics: Fundamentals, Modelling, Applications.* , Springer-Verlag, New York, (2008).

[2] J. B. Pendry, A. J. Holden, D. J. Robbins, W. J. Stewart, *Magnetism from conductors and enhanced nonlinear phenomena*, IEEE Trans. Microwave Theory Tech, **47**, 2075, (1999).

[3] R. A. Shelby, D. R. Smith, S. C. Nemat-Nasser, S. Schultz, *Microwave transmission through a two-dimensional, isotropic, left-handed metamaterial*, Appl. Phys. Lett., **78**, 489 (2001); R. A. Shelby, D. R. Smith, S. Schultz, *Experimental Verification of a Negative Index of Refraction*, Science, **292**, 77 (2001).

[4] M. C. Ricci and S. M. Anlage, *Single superconducting split-ring resonator electrodynamics*, Appl. Phys. Lett. **88**, 264102 (2006).

[5] M. C. Ricci, N. Orloff, S. M. Anlage, *Superconducting metamaterials*, Appl. Phys Lett. **87**, 034102 (2005).

[6] C. Kurter, J. Abrahams, S. M. Anlage, *Miniaturized Superconducting Metamaterials for Radio Frequencies*, Appl. Phys. Lett. **96**, 253504 (2010).

[7] C. Kurter, A. P. Zhuravel, J. Abrahams, C. L. Bennett, A. V. Ustinov, S. M. Anlage, *Superconducting RF Metamaterials Made with Magnetically Active Planar Spirals*, IEEE Trans. Appl. Supercond. **21**, 709 (2011).

[8] S. Kh. Kogan, *Rasprostranenie voln vdol beskonechnoi spirali*, DAN, **66**, 867 (1949) [in Rq ussian].

[9] R. A. Silin and V. P. Sazonov, *Zamedlyaushie Sistemi*, Sov. Radio, Moscow (1966) [in Russian].