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Effects of Energy Dissipation on the Parametric Excitation of a Coupled Qubit–Cavity System

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Abstract We consider a parametrically driven system of a qubit coupled to a cavity taking into account different channels of energy dissipation. We focus on the periodic modulation of a single parameter of this hybrid system, which is the coupling constant between the two subsystems. Such a modulation is possible within the superconducting realization of qubit–cavity coupled systems, characterized by an outstanding degree of tunability and flexibility. Our major result is that energy dissipation in the cavity can enhance population of the excited state of the qubit in the steady state, while energy dissipation in the qubit subsystem can enhance the number of photons generated from vacuum. We find optimal parameters for the realization of such dissipation-induced amplification of quantum effects. Our results might be of importance for the full control of quantum states of coupled systems as well as for the storage and engineering of quantum states.

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1 Introduction

Superconducting quantum circuits are characterized by a high degree of tunability and flexibility [1–6]. In such systems, the populations of superconducting qubits and microwave resonators can be affected either directly [7–9] or by parametric modulation [4]. This makes it possible to use superconducting Josephson circuits for the exploration of nonadiabatic quantum electrodynamics phenomena, which are hard to implement in other systems, see, e.g., Refs. [10–13].

Moreover, the outstanding flexibility of superconducting Josephson circuits is utilized in quantum computation for the construction of parametric gates, which have certain advantages compared to the more standard approach based on fixed-frequency qubits with static coupling suffering from cross talks [14]. Instead, one can either rely on frequency-tunable transmons, which however results in the appearance of new decoherence channels and frequency crowding [14,15], or on parametrically modulating couplings between different constituent parts of the whole system which has the advantage of a high degree of selectivity [14,16–19]. Thus, investigation of parametrically driven circuits is of importance for both fundamental science and applications.

In our recent papers [13,20], we studied a coupled system of a superconducting qubit coupled to a microwave resonator with a modulated coupling constant between these two subsystems. We argued that such a system can be used to realize in a nontrivial way the dynamical Lamb effect [21], which can be treated as parametric qubit excitation due to the nonadiabatic modulation of its dressing by virtual photons. Moreover, this effect can be quite strong provided the qubit and the cavity are in resonance. The effect of energy dissipation on the dynamics of such a system is also highly nontrivial [20]—quantum effects in the photon subsystem can be enhanced by the finite energy dissipation in the qubit subsystem.

In the present article we study more systematically this remarkable phenomenon by carefully analyzing the steady state of the system, which depends on several controlling parameters. We show that the effect is rather general—quantum effects in the qubit subsystem, i.e., the population of the excited state of the qubit due to parametrical processes can be also enhanced by the finite cavity decay. We then reveal the optimal parameters to achieve such an enhancement within both channels. Our findings can be used to provide the full control of the qubit-resonator quantum states.

2 Hamiltonian and Basic Equations

We consider a coupled system consisting of a superconducting qubit and single-mode quantum resonator. Within the quantum optics framework, the Hamiltonian of this hybrid system reads as

$$H(t) = \omega a^\dagger a + \epsilon_j \sigma^+ \sigma^- + G(t)(\sigma^+ + \sigma^-)(a^\dagger + a), \qquad (1)$$

where $a^\dagger$ and a are secondary quantized photon creation and annihilation operators, σ^+, σ^- are Pauli operators acting in the subspace of qubit degrees of freedom, while $G(t)$ is the interaction constant between qubit and cavity, which is dynamically tunable. Dynamical tunability can be achieved using different approaches [22–24]. For instance, it is possible to use a three-level quantum system (transmon) under coherent drive, which might behave like an effective two-level system with tunable coupling to the cavity [24]. Another method is based on two strongly interacting transmons with hybridized energy levels [23]. The lowest excited level is more weakly coupled to light due to the symmetry of the total wave function, which becomes purely antisymmetric and therefore fully decoupled from the photon field in the case of resonance. This lower excited level is then treated as a first excited state of a logical qubit. By tuning independently the transmon's frequencies, it is possible to tune the coupling of the logical qubit to the microwave tone, while keeping fixed the excitation frequency.

The last term on the right-hand side of Eq. (1), which describes the interaction between the subsystems, contains the two contributions known as the rotating wave term, $G(t)V_1 = G(t)(\sigma^+ a + \sigma^- a^\dagger)$, and the counterrotating wave term, $G(t)V_2 = G(t)(\sigma^+ a^\dagger + \sigma^- a)$. For stationary systems, the counterrotating term can be neglected near the resonance provided the interaction constant is much smaller than the cavity frequency. However, under periodic modulation of the interaction constant, it must be kept; moreover, it can be essential for a correct description of the system's dynamics [25–30].

Decoherence effects are accounted for by solving numerically the Lindblad equation

$$\partial_t \rho(t) - \Gamma[\rho(t)] = -i[H(t), \rho(t)], \tag{2}$$

where $\rho(t)$ is the density matrix of the coupled qubit and photon subsystems. The matrix $\Gamma[\rho]$ depends on the rates of energy dissipation in the cavity κ, in the qubit γ, as well as on the pure dephasing rate γ_φ. It is given by

$$\Gamma[\rho] = \kappa \left(a\rho a^\dagger - \frac{1}{2}\{a^\dagger a, \rho\} \right) + \gamma \left(\sigma^- \rho \sigma^+ - \frac{1}{2}\{\sigma^+ \sigma^-, \rho\} \right) + \gamma_\varphi \left(\sigma^z \rho \sigma^z - \rho \right).$$

We assume that initially the qubit and the cavity are uncoupled and that both of them are in their ground states, $|\downarrow, 0\rangle$. Then, the periodic modulation of the coupling constant is turned on. When the qubit and the cavity are in resonance, the effect of such a modulation is strongest. It takes place when the modulation frequency is twice the cavity frequency [13,20], leading to parametric resonance. Theoretical analysis of this particular situation can be significantly simplified by using a separation in fast and slow degrees of freedom and performing time averaging. This is due to the presence of two types of oscillations in the dynamics of our system—small-amplitude and fast oscillations with the frequency of the photon field and large-amplitude oscillations with much smaller frequencies of the order of the Rabi frequency. The fast and insignificant oscillations are eliminated by a time averaging procedure [13,20], making numerical solution of the master equation much simpler. In this case and in the limit $|G(t)| \ll \omega$, the system's dynamics is controlled by two Fourier components [13,20] defined as $\langle G(t)\rangle_t \equiv p$ and $\langle G(t)\exp(-2i\omega t)\rangle_t \equiv q$. We represent p and q as $p = g\theta$,

$q = g(1 - \theta)$, where $0 \leq \theta \leq 1$. The modulation signal is sign alternating at $q > p$ ($\theta < 0.5$) and nonsign alternating at $q < p$ ($\theta > 0.5$), while $\theta = 1/2$ corresponds to the situation when both components are the same and the signal having only these two components vanishes periodically in time, but does not change its sign. The parameter p controls the strength of the interaction in the Tavis–Cummings channel, which conserves the excitation number, while q is responsible for the counterrotating processes which change the excitation number by ± 2 [13,20].

We thus treat the Hamiltonian in the interaction picture with the coupling between the qubit and photon subsystems given by $g[\theta V_1 + (1-\theta)V_2]$, which is not dependent explicitly on time, where $V_1 = \sigma^+ a + \sigma^- a^\dagger$ is the interaction in the Tavis–Cummings channel and $V_2 = \sigma^+ a^\dagger + \sigma^- a$ is the counterrotating term. We then analyze the dynamics after sudden switching of the coupling constant from zero to g, the initial state of the system being $|\downarrow, 0\rangle$. We focus on the steady state and address the behavior of both the population of the qubit's excited state and the mean number of photons generated from vacuum in the space of four parameters $(g, \theta, \gamma = \gamma_\varphi, \kappa)$. In principle, γ_φ is also an independent parameter. However, we use a fixed constraint $\gamma = \gamma_\varphi$, while, according to our results, tuning γ_φ independently does not significantly alter our conclusions as long as the condition $\gamma_\varphi \sim \gamma$ is satisfied.

3 Steady State: Population of the Excited State of the Qubit

In this section, we focus on the steady-state population n_q of the excited state of the qubit, which is achieved in the stationary state after the system's evolution. We would like to stress that the qubit is excited parametrically from its ground state by the counterrotating term of the Hamiltonian. Our aim is to study the influence of energy dissipation in the photon subsystem on the population of the qubit's excited state.

Figure 1 shows the color map of the population of the qubit's excited state n_q in the plane of cavity dissipation rate κ and parameter θ at three different values of qubit dissipation rate and pure dephasing rate $\gamma = \gamma_\varphi$ and at fixed value of coupling constant amplitude $g = 0.05\omega$. Notice that, for illustrative purposes, we have chosen a rather high value of g, but our major conclusions are not so sensitive to the ratio g/ω as long as it remains small, as explained below. The analysis of Fig. 1 as well as the inspection of similar maps for other values of g/ω leads to the counterintuitive conclusion that the largest effect of parametric qubit excitation in the steady state is achieved not in the case of an ideal cavity without losses, but in the cavity with some optimal and nonzero decay rate κ_opt, and always at $\theta = 0$. The latter condition implies full domination of the counterrotating processes over the excitation-number conserving dynamics. The optimal cavity decay rate is determined by the condition $\kappa_\mathrm{opt}\gamma \approx g^2$, which is deduced from the analysis of maps at different values of g.

It is also clear from Fig. 1 that the effect of qubit relaxation γ on the quantum effects in the same subsystem is standard: It leads to the suppression of the population of the qubit's excited state.

The fact that n_q is maximum at $\theta = 0$ can be understood by noting that the parameter q is responsible for the efficiency of counterrotating processes, i.e., for the parametric excitation of qubit from its ground state ($|\downarrow, 0\rangle \rightarrow |\uparrow, 1\rangle$). Therefore, full domination

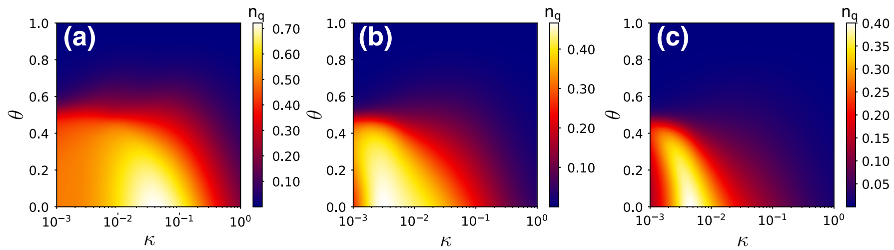

Fig. 1 Color map for the population of the qubit's excited state in steady-state conditions in the plane of cavity dissipation rate κ and parameter θ at $g = 0.05\omega$ and three different values of the qubit's dissipation rate γ: $\gamma = 0.01\omega$ (**a**), $\gamma = 0.05\omega$ (**b**) and $\gamma = 0.1\omega$ (**c**); $\gamma_\varphi = \gamma$ in all cases (Color figure online)

of counterrotating processes over excitation-number conserving dynamics is achieved at $\theta = 0$. It is also clear that relaxation in the qubit yields an opposite effect, and hence n_q is highest at $\gamma = 0$. However, the effect of relaxation in a cavity is much more nontrivial. Actually, it creates a new channel of system relaxation from $|\uparrow, 1\rangle$ to the initial state $|\downarrow, 0\rangle$ through the intermediate state $|\uparrow, 0\rangle$, the latter containing the qubit in its excited state. Thus, the presence of both relaxation channels (in the qubit and in the cavity) allows to create a kind of a circle $|\downarrow, 0\rangle \rightarrow |\uparrow, 1\rangle \rightarrow |\uparrow, 0\rangle \rightarrow |\downarrow, 0\rangle$, which leads to the partial trapping of the qubit in its excited state.

4 Steady State: Mean Photon Number

Let us now analyze the mean photon number n_{ph} as a function of controlling parameters in the steady state, with particular emphasis on the influence of dissipation in the qubit subsystem. The photons are generated from vacuum by the counterrotating term, which is responsible for the simultaneous qubit excitation and photon production.

Figure 2 provides color maps for the mean photon number n_{ph} in the steady state in the plane of qubit dissipation rate $\gamma = \gamma_\varphi$ and parameter θ at three different values of cavity dissipation rate κ and at $g = 0.05\omega$. We see that the evolution of n_{ph} is rather nontrivial. At low $\kappa \lesssim g$, the maximum n_{ph} is achieved at $\theta = 0.5$ and $\gamma = 0$. This regime is, in some sense, normal, since it indicates that the smaller the energy dissipation, the stronger the quantum effects. In this case, photon production is dominated by coherent processes V_1 and V_2 responsible for the transitions in the ladder of bare states of the form $|\downarrow, 0\rangle \rightarrow |\uparrow, 1\rangle \rightarrow |\downarrow, 2\rangle \rightarrow |\uparrow, 3\rangle \rightarrow \ldots$. Such transitions lead to the production of photons from vacuum, and the maximum efficiency of this production is achieved at $\theta = 0.5$, i.e., when V_1 and V_2 are equally efficient.

Figure 2a also shows that another local maximum of n_{ph} does exist at a different value of θ equal to 1 and at finite γ, the two maxima in the map being connected by a kind of an arc with locally enhanced n_{ph}. As κ grows and approaches g, this local maximum transforms into a global maximum, the transition being discontinuous. Thus, the enhancement of photon generation from vacuum due to the energy dissipation in the qubit subsystem starts dominating at nonzero threshold cavity relaxation $\approx g$. The analysis of maps at different values of g also puts in evidence that the optimal γ_{opt} which maximizes n_{ph} in this regime is also nearly equal to g. The existence of this

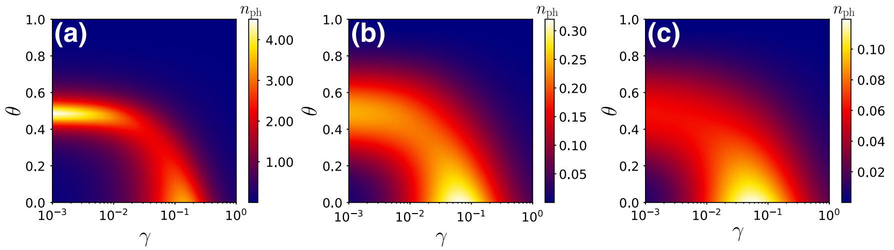

Fig. 2 Color map for the mean photon number in the steady state in the plane of qubit dissipation rate γ and parameter θ at $g = 0.05\omega$ and three different values of cavity dissipation rate κ: $\kappa = 0.01\omega$ (**a**), $\kappa = 0.05\omega$ (**b**) and $\kappa = 0.1\omega$ (**c**) (Color figure online)

nontrivial regime is linked to the suppression of coherent generation of photons from vacuum through V_1 and V_2, which is more efficient at low κ. It is quite natural that photon decay suppresses such a generation. However, qubit relaxation opens a new channel for the system excitation. Indeed, V_2 is able to excite the system only provided the qubit is in its ground state. Thus, qubit de-excitation due to the finite γ yields a new set of transitions of the form $|\downarrow, 0\rangle \to |\uparrow, 1\rangle \to |\downarrow, 1\rangle \to |\uparrow, 2\rangle \to \ldots$, which enhance the mean number of photons in the cavity. Such a dynamics is dominated by V_2 and qubit relaxation, while V_1 becomes not so important. Therefore, maximum photon number is achieved at $\theta = 0$.

The maximum of n_{ph} at $\theta = 0.5$, which does not exist for n_q at the same θ, indicates a clear asymmetry between the qubit and the cavity in our system. The asymmetry originates from the two-level character of the qubit and from the structure of counterrotating term V_2, which can produce arbitrarily large number of excitations in the cavity, but no more than a single excitation in the qubit.

The inspection of Fig. 2 also proves that the effect of cavity relaxation κ on quantum effects in the cavity is quite expected, i.e., when it takes place in the same subsystem, since it leads to the suppression of the mean photon number generated from vacuum.

5 Discussion and Summary

In the present article, we studied theoretically a parametrically driven system consisting of a single qubit coupled to a cavity. Periodical parametric modulation of the coupling constant leads to amplification of counterrotating processes accompanied by qubit excitation with simultaneous photon creation from vacuum. Qubit–cavity systems with dynamically tunable couplings can be engineered with superconducting realization using various schemes.

Our main conclusions are as follows:

(a) Energy dissipation in such a system leads to rather unexpected dynamics—quantum effects in any of the two subsystems of a hybrid system can be significantly enhanced due to the energy dissipation in the other subsystem. This happens because the energy dissipation in such a multilevel system is able to open new channels of system's dynamics.

(b) There exists an optimal nonzero cavity dissipation rate κ_{opt}, which maximizes the population of the qubit's excited state in the steady state, at a given qubit relaxation rate γ of the qubit. The maximum population is achieved for a modulation signal, which enhances counterrotating processes. The relation between κ_{opt}, γ and the maximum of the qubit–cavity coupling constant g is $\kappa_{\text{opt}}\gamma \approx g^2$.
(c) For the maximum mean photon number in the steady state there exist two regimes at a given cavity relaxation rate κ. In the first case, maximum photon number is achieved at the qubit relaxation rate γ equal to zero. This situation is realized at small values of the cavity relaxation rate $\kappa \lesssim g$. It corresponds to the modulation signal, which supports equal efficiency of excitation-number conserving and counterrotating terms. In the second case, the maximum photon number is achieved at nonzero qubit relaxation rate $\gamma_{\text{opt}} \approx g$ and for the sign-alternating signal. This unusual regime is realized at relatively large values of the cavity relaxation rate $\kappa \gtrsim g$. The transition between the two regimes is discontinuous.
(d) The unconventional effect arising from the enhancement of quantum effects in one of the subsystems by energy dissipation in the other subsystem is definitely linked to the domination of counterrotating processes, since it always appears in the domain of parameters of the modulation signal supporting such a domination.
(e) In contrast, the influence of energy dissipation on quantum effects in the same subsystem is conventional—energy dissipation suppresses such phenomena.

Our results provide deeper insights into the physics of parametrically driven quantum circuits. They might be of importance for dissipative quantum computation as well as for the storage and protection of quantum states.

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