Superconducting Split-Ring Resonators with Embedded Tunable Inductors

A. Vidiborskiy [1], V. P. Koshelets [1,3], L. V. Filippenko [1,3], S. V. Shitov [1,3] and A. V. Ustinov [1,2]

[1] National University of Science and Technology (MISIS), Leninsky prosp. 4, Moscow 119049, Russia
[2] Physikalisches Institut, Karlsruhe Institute of Technology (KIT), 76131 Karlsruhe, Germany
[3] Kotel'nikov Institute of Radio Engineering and Electronics (IREE RAS), Moscow 125009, Russia
e-mail of the presenter: vidiborskiy@gmail.com

Abstract – **We propose tunable superconducting split-ring resonators (SRRs) employing nonlinear inductance. A fraction of SRR is replaced by Nb-AlO$_x$-Nb Josephson tunnel junctions connected in parallel and forming a superconducting quantum interference device (SQUID), which inductance is sensitive to the external magnetic field. Due to this modification, the SRR can be made smaller and its resonance frequency can be tuned via application of magnetic field. To check the effect of tuning, the resonator is coupled to a microstrip line, which transmission is dependent on the resonant frequency of the SRR. We present the practical layout and results of extensive EM-simulation for transmission coefficient S$_{21}$ within frequency range 8-12 GHz.**

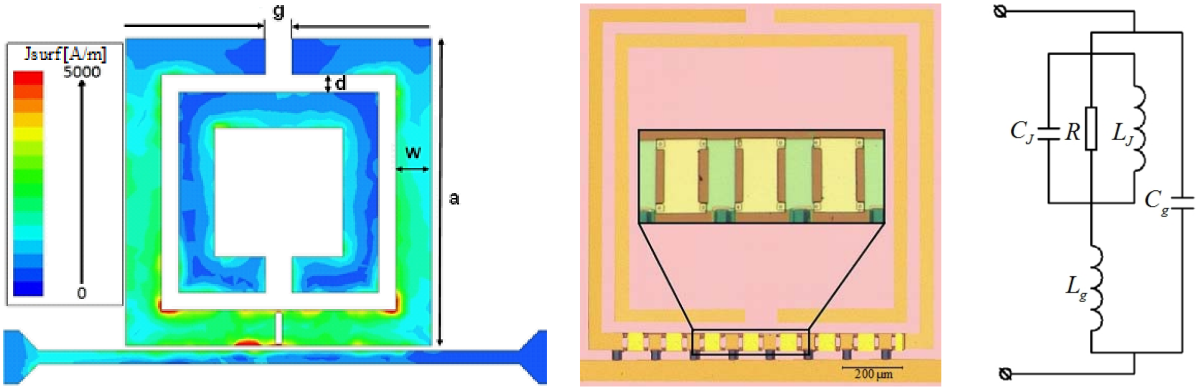
I. INTRODUCTION

Nowadays, metamaterials offer a wide range of possible applications: cloaking, super lenses, solar energetics, *etc*. One of the interesting areas includes metamaterials with negative index of refraction that requires simultaneously negative permittivity and negative permeability. To construct such a media, it is necessary to design special elements, which react with the magnetic and electric components of the incident wave. The split-ring resonators (SRRs) are often used for tailoring the magnetic reaction of the medium and, in general, have resonance frequency that is determined by linear dimensions of the rings.

In recent studies [1, 2] different approaches to making tunable split-ring resonators have been proposed. Most conventional way is by using a semiconductor varactor as a tunable nonlinear capacitance. The varactor can be soldered directly to the ring. Such combination works in negative permeability regime around resonance frequency, which can be shifted, for example, with infrared light [1]. Here, we propose using superconducting SRRs with integrated Josephson junctions (JJ) forming the chain of SQUIDs. This approach makes meta-atoms very compact and provides a possibility of tuning the resonance frequency with an external magnetic field. In this case, the inductance of the SRR is a periodic function of the total magnetic flux threading the SQUIDs.

II. LAYOUT AND SIMULATIONS

To facilitate the analysis of the SRR, we use the approximation of a lumped *LC*-circuit [3]. In this case, the inductance *L* is defined by inductance per unit of length of two parallel wires and by their mutual inductance. The full capacitance of the circuit, *C*, is given by the sum of two components, the capacitance of the gap and the capacitance between two plates separated by a dielectric. As a reference we used the following SRR parameters (Fig. 1a): $a = 1.34\,\text{mm}, g = 0.12\,\text{mm}, d = 0.12\,\text{mm}, w = 0.14\,\text{mm}, \varepsilon = 11.9, h_{sub} = 0.3\,\text{mm}$. In this case, the resonance frequency, calculated from the above model, is $f_{res} = 11.48\,\text{GHz}$. The SRR with embedded JJ have been simulated using the AWR MWO environment [4]. The resonators were placed aside a 50-Ohm microstrip line. To enhance the coupling, the transmission line is made narrower within the coupling region. Since SRR effectively is a semi-lumped resonator, the antinode of the standing wave will be located in the center of the outer ring, as seen in Fig. 1a. Due to symmetry and the principle of minimal energy, a node of the standing wave is located at the ends of outer ring that corresponds to the minimum of the surface current. Considering the distribution of the current in the resonator, the most effective position for introducing an extra inductor is the area of maximum of rf current, in the middle of the outer part of the SRR (Fig. 1a, Fig. 1b).



a) b) c)

Fig. 1. a) The distribution of the surface current in SRR with single dc-SQUID. b) Photo of experimental silicon chip with SRR containing 15 dc-SQUIDs. c) Equivalent electrical scheme of the SSR loaded with single JJ (similar to case a)).

Since the MWO EM-simulator does not support direct implementation of the Josephson effect, we use the electrical equivalent of a JJ. The linear model of a JJ can be presented as parallel connection of a leaky capacitor, which substitutes the tunnel barrier, and an H-field dependent inductor representing the Josephson inductance. The Nb-AlO$_x$-Nb Josephson junctions are fabricated with the following parameters: $S = 7 \mu m^2$, $R_{na} = 2000\,\text{Ohms} \cdot \mu m^2, \Delta = 2.2\,\text{mV}$. The critical current of the junction can be estimated with the Ambegaokar-Baratoff relation [5]:

$$I_c^{A.-B.} = \frac{\pi\Delta}{2eR_n}, \tag{1}$$

here $I_c^{A.-B.}$ is the critical current, R_n is the normal state resistance, and Δ is the superconducting energy gap of Nb. The maximum superconducting current through the junction can be estimated as $I_c^{A.-B.} = 10.35\,\mu A$. However, the experimental critical current is usually smaller, typically about 25% of the value predicted by (1), i.e. about $I_c \approx 2.5\ \mu A$. The variable Josephson inductance is given by expression:

$$L_J = \frac{\Phi_0}{2\pi I_c \cos\varphi}, \tag{2}$$

where Φ_0 is the magnetic flux quantum, $\cos\varphi$ – the cosine of superconducting phase difference across the junction, which is unity in zero magnetic field. The following parameters can be expected for the Josephson junctions [6]: $C_J = 490\,\text{fF}, L_J^{min} = 0.13\,\text{nH}, L_J^{max} = 0.27\,\text{nH}$. The equivalent circuit for the SRR embedded with single SQUID is presented in Fig. 1c. Since we have chosen to keep the operating range near 11.5 GHz, it is necessary to reduce the linear size of SRR, which requires increasing the number of SQUIDs. The effect of reducing the linear size of SRR at cost of added SQUIDs is presented in Table 1.

Table 1. Parameters of SSR with different number of SQUIDs

Number of SQUIDs	SRR dimension *a*, mm	Tuning range, GHz
0	1.38	11.45 GHz, not tunable
1	1.38	11.05 ... 11.35
3	1.22	10.7 ... 11.45
9	1.02	10.15 ... 11.65
15	0.82	9.7 ... 11.65

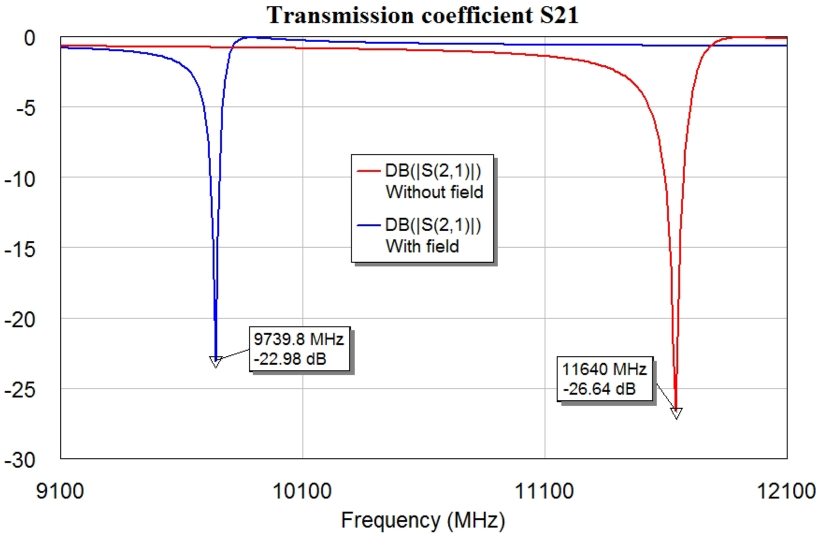


Fig. 2. Simulated transmission coefficient S_{21} of the microstrip-coupled SRR with 15 embedded SQUIDs.

The results of modeling are presented in Fig. 2. The resonant dip has the highest frequency at zero magnetic field and shifts to the lowest frequency at half magnetic flux-quantum applied to the SQUIDs. According to this simulation, it should be possible to shift the operating frequency of this SRR by approximately 2 GHz.

III. Conclusions

A fully integrated tunable split-ring resonator loaded with Josephson junctions is proposed and studied numerically. This approach provides a possibility to scale down the SRR dimensions while retaining the resonance frequency. We find that the size of SRR with 15 SQUIDs can be reduced by a factor 1.7 maintaining the same zero-field resonance frequency of about 11.5 GHz. Variation of the external dc magnetic field gives a possibility to change the resonance frequency of the SRR by about 20% or $\Delta f_{res} \approx 2\,\text{GHz}$. Preliminary experimental data are already available and will be presented at the conference; a reasonable consistency with the simulations is demonstrated.

Acknowledgement

This work was supported in part by the Ministry of Education and Science of the Russian Federation.

References

[1] K. A. Boulais, D. W. Rule, S. Simmons, F. Santiago, V. Gehman, K. Long and A. Rayms-Keller, "Tunable split-ring resonator for metamaterials using photocapacitance of semi-insulating GaAs," *Appl. Phys. Lett.,* vol. 93, p. 043518, 2008.
[2] I. Gil, J. Garcia-Garcia, J. Bonache, F. Martin, M. Sorolla and R. Marques, "Varactor-loaded split ring resonators for tunable notch filters at microwave frequencies," *Electronics Letters*, vol. 40, pp. 1347-1348, 2004.
[3] M.R. Vidyalakshmi and Dr. S. Raghavan, "Comparison of Optimization Techniques for Square Split Ring Resonator," *IJMOT*, vol. 5, pp. 280-286, 2010.
[4] http://www.awrcorp.com/products/microwave-office
[5] T. van Duzer and C.W. Turner, *Principles of superconductive devices and circuits*, 2nd ed., Upper Saddle River, NJ, USA: Prentice Hall PTR, 1999.
[6] http://www.hypres.com/wp-content/uploads/2010/11/DesignRules.pdf