Analysis and Design of Superconducting Left-Handed Transmission Lines

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***Abstract*—In this paper, for the first time, we study the properties of a superconductive-based left-handed transmission line (SLHTL). The effect of the kinetic inductance on the propagation constant of the left-handed transmission line is analyzed. It is shown that increasing the kinetic inductance decreases both the bragg frequency and the negative index frequency band of a distributed LHTL. A Coplanar waveguide structure is proposed to realize the LHTL. It is shown that the CPW structure has the capability of increasing the kinetic inductance of the TL by narrowing the spacing between the line and the ground plane because they are deposited on the same side of the substrate. This was not possible in a microstrip line as the spacing between the line and the ground is equal to the substrate thickness and is usually fixed. Furthermore, the dependency of the kinetic inductance to the bias current has been employed to design a tunable SLHTL. Possible applications of the tunable SLHTL in the design of miniaturized tunable resonators and filters are discussed.**

I. Introduction

THE concept of negative index materials and left-handed transmission lines have recently attracted Physics and Engineering community [1]–[4]. Negative index materials is referred to artificially engineered periodic structures that simultaneously exhibit negative effective permittivity and permeability [5], [6]. A good review article on the subject can be found in [7]. These artificial electromagnetic materials exhibit extraordinary properties, e.g. the direction of the energy propagation is in the opposite direction of phase propagation for a plane wave traveling in negative index material. Therefor, the $[\vec{E}, \vec{H}, \vec{k}]$ makes a left-handed triplet and the name left-handed transmission line is stemmed from this fact.

The concept of a material with negative permittivity and permeability was first studied by Veselago in 1968 [8]. It was not until recently that different groups tried to artificially realize a structure that exhibits negative effective permittivity and permeability. Different groups designed periodic structures of split ring resonators and wires that show such a properties over a narrow frequency band [9]. Other groups proposed a transmission line model, which is referred to as Left-handed TL, by changing the locations of the inductors and capacitors of the traditional transmission line model [10], [11]. This results in a dispersive transmission line structure that exhibits negative permittivity and permeability over a wide frequency range. The structure is implemented by loading a transmission line with series capacitors and shunt inductors.

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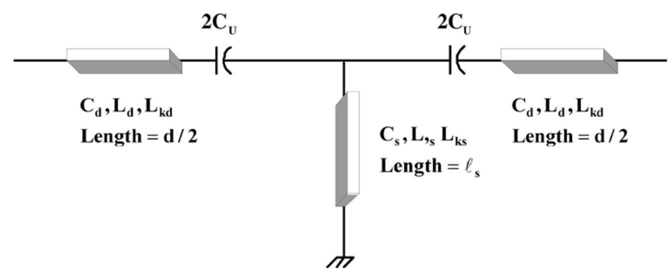

Fig. 1. Circuit model of a distributed LHTL unit cell.

In this paper, we study the effect of incorporating superconductor in the structure of a LHTL. As expected, the use of superconductor drastically diminish the power dissipation in the system. We also study the effect of kinetic inductance on the index of refraction of the SLHTL. To choose a host transmission line structure, the kinetic inductance of microstrip and CPW lines are compared. It is shown that the effect of kinetic inductance is increased if the spacing between the line and the ground plane is decreased. CPW line offers the possibility of increasing its kinetic inductance by decreasing the spacing between the line and the ground plane while in the microstrip structure the spacing between the line and the ground plane is fixed. Therefore the CPW line is chosen to realize the SLHTL. A tuneable SLHTL is proposed where the shunt inductor is controlled by an external DC bias current.

II. LHTL Structure

The concept of a left-handed transmission line (LHTL) has been studied in the literature [10]–[12]. The left-handed transmission line has two important characteristics. First and foremost, its propagation constant, β, is negative. This corresponds to the fact that in a LHTL, the direction of phase propagation and energy propagation are anti-parallel. The second property of the LHTL model is its dispersiveness, even when it operates in the lossless regime, i.e. $\beta \propto 1/\omega$.

The lumped element model of a distributed LHTL unit cell and its physical implementation in coplanar technology are depicted in Fig. 1 and Fig. 2, respectively. The capacitor is realized by an interdigital capacitor and the inductor is modeled by a short stub meander line. The propagation constant, β, of an array of distributed L-C unit cells, depicted in Fig. 3, can be determined following the standard analysis of 1-D periodic struc-

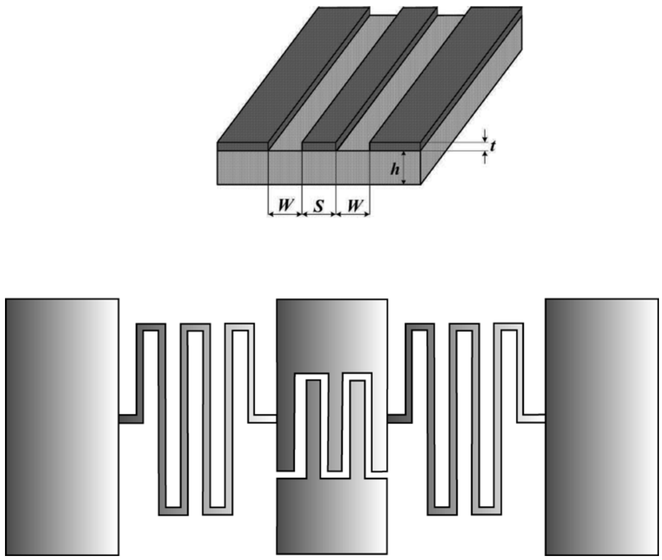

Fig. 2. Implementation of a LHTL unit cell using CPW TL.

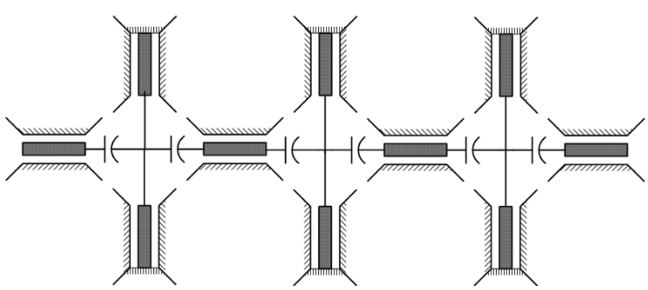

Fig. 3. Distributed LHTL using CPW.

tures presented in [1]. The dispersion relation of this structure obtained in [1], is given by

$$\begin{aligned}\cos(\beta d) &= \cos(\omega\sqrt{L_d C_d}d)\\ &\quad - \frac{1}{2}\left[\omega\sqrt{\frac{C_s}{Ls}}C_L \cot(\omega\sqrt{L_s C_s}\ell_s)\right]\\ &\quad \times \cos^2\left(\frac{\omega\sqrt{L_d C_d}d}{2}\right)\\ &\quad + \frac{1}{2}\left[\frac{\omega C_L}{\sqrt{\frac{L_d}{C_d}}} + \sqrt{\frac{C_s L_d}{C_d L_s}}\cot(\omega\sqrt{L_s C_s}\ell_s)\right]\\ &\quad \times \sin(\omega\sqrt{L_d C_d}d) \qquad (1)\end{aligned}$$

where L_d and C_d are the inductance and capacitance per unit length of the transmission line that connects the interdigital capacitor and the short stub. L_s and C_s are the inductance and capacitance per unit length of the short stub. d is the unit cell length and ℓ_s is the short stub length. The dispersion diagram of a distributed LHTL with $L_d = 533\ \mathrm{nH/m}$, $C_d = 104\ \mathrm{pF/m}$, $L_s = 527\,\mathrm{nH/m}$, $C_s = 105\,\mathrm{pH/m}$, $d = 3\,\mathrm{mm}$, and $\ell_s = 3\,\mathrm{mm}$ is depicted in Fig. 4. The distributed LHTL operates as a LHTL in a frequency range above the brag frequency, f_b, and below the first cut off frequency, f_{c1}, i.e. $[f_b, f_{c1}]$. The line behaves as a traditional RHTL for frequencies above f_{c2}. The frequency response of such a distributed LHTL is shown in Fig. 5. The

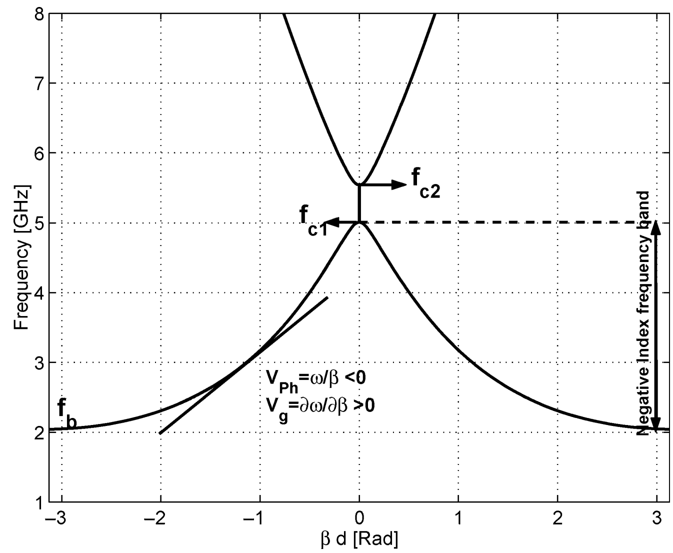

Fig. 4. Dispersion diagram of the distributed LHTL.

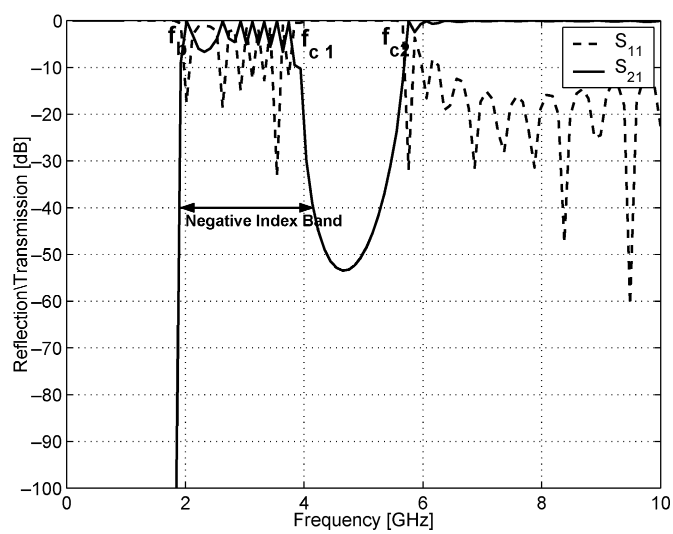

Fig. 5. S-parameters of the left-handed transmission line consisting of 20 unit cells.

interdigital capacitor is estimated to be 0.6 pF and the shunt inductor is set at 6.7 nH. The frequency band in which the structure exhibits left-handedness is marked in the figure, with f_b, f_{c1}, and f_{c2} corresponding to those in the dispersion diagram depicted in Fig. 4. The cut off frequencies indicated in Fig. 5, f_{c1} and f_{c2}, are slightly different from those drawn in Fig. 4. This is due to the fact that in the frequency response analysis, instead of using the distributed shunt inductor, a lumped element inductor is employed.

III. CPW-BASED SLHTL

In this section, we study a LHTL realized with superconductive materials. The performance of any device that is built using good conductors to carry current can be improved simply with the substitution of the superconductors in the current pass. This also increases the total inductance per unit length of the transmission line. To implement a LHTL, it is necessary to choose one of the traditional transmission line structures, e.g. microstip

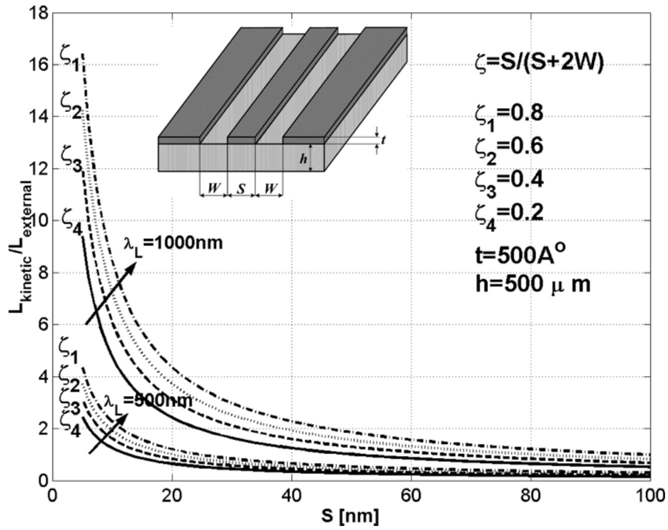

Fig. 6. The ratio of the kinetic inductance to external inductance of a coplanar waveguide as a function of its strip width.

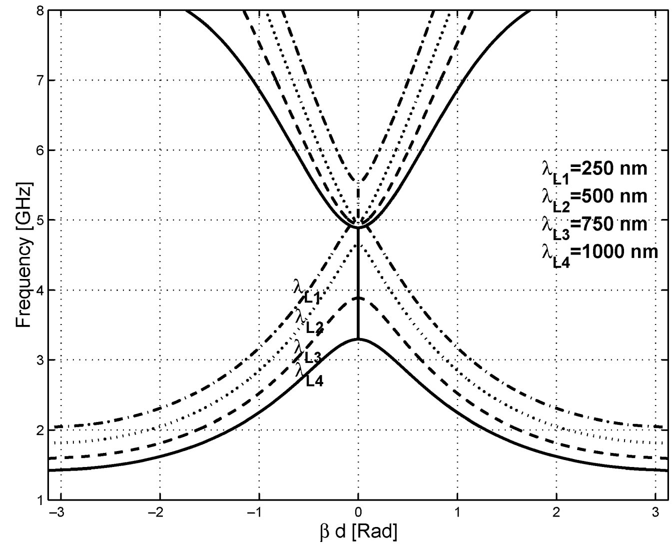

Fig. 7. Dispersion diagram of SLHTL with different penetration depths. $S_{shunt} = 10\,\mu\mathrm{m}, W_{shunt} = 10\,\mu\mathrm{m}, S_{Line} = 1.5\,\mathrm{mm}, W_{Line} = 2.0\,\mathrm{mm}$.

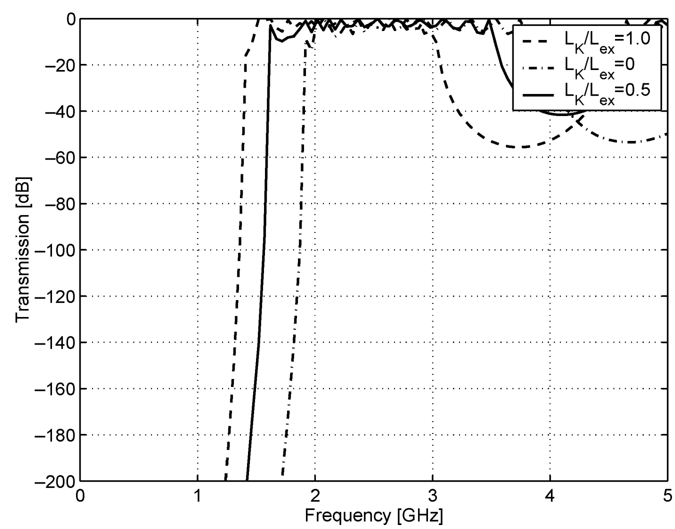

Fig. 8. S-parameters of the LHTL as kinetic inductance of the shunt inductor varies.

line, strip line, or CPW line. The distributed LHTL, described in [11], is realized using a Microstrip TL. Microstrip TL is one of the most popular industry standards and a large amount of information about it is available. However, the effect of kinetic inductance in the total inductance per unit length of the microstrip line is quite weak. This can be explained by examining the analytical expression for the inductance per unit length of the wide microstrip line presented in [12]. The inductance per unit length of a wide microstrip line derived in [12] can be written as $(\mu_0/W)(h + 2\lambda \coth(t/\lambda))$ where W is the width of the line, t is the thickness, h is the substrate thickness, and λ is the penetration depth. For a typical structure, the substrate thickness is in the range of 500 $\mu\mathrm{m}$ while the penetration depth, λ, is normally less than 1 $\mu\mathrm{m}$. Therefore, the kinetic inductance of the microstrip line is much smaller than its external inductance. One way to increase the effect of the kinetic inductance is to decrease the substrate thickness. Usually, reducing the substrate thickness is not practical. The alternative way to increase the effect of the kinetic inductance is to use a coplanar waveguide structure instead of the microstrip line. Employing the coplanar waveguide structure makes it possible to decrease the distance between the line and the ground plane without changing the substrate thickness. The ratio of the kinetic inductance to external inductance of a coplanar waveguide as a function of its strip width is depicted in Fig. 6 using the analytical expressions given in [13]. Employing CPW lines instead of microstrip line has also the advantages that it only requires a one-sided deposition of superconductor material and the need to use via holes to connect the shunt stubs to the ground plane is removed.

As it can be observed from Fig. 6, changing the penetration depth of the superconductor material changes the kinetic inductance of the line which in turns changes the total inductance per unit length of the line. This specially affects the inductance of the shunt stub as it is narrower than the main line and therefore the effect of kinetic inductance is more significant in its total inductance. Fig. 7 shows that increasing the penetration depth decreases the bragg frequency, f_b, and reduces the negative index frequency band of a LHTL. Fig. 7 also shows that for a fixed frequency, the propagation constant of the LHTL varies as penetration depth varies. This implies that the index of refraction of the line is a function of the penetration depth. In the next section, we discuss how employing an external DC bias current changes the kinetic inductance of the structure and can result in a tunable LHTL.

IV. Tunable LHTL

The presented analysis showed that incorporating superconductor in the LHTL structure induces the kinetic inductance that can change the properties of a LHTL when the kinetic inductance is considerably large in comparison with the external inductance of the line (Fig. 8). In this section, we show that how employing a DC bias can control the kinetic inductance of the LHTL line, which results in a tunable LHTL. Application of a DC current bias, through the use of a proper bias circuit, will change the surface kinetic inductance, i.e., $L_{Ksurface} =$

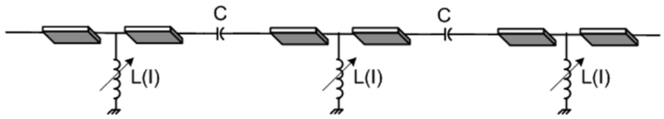

Fig. 9. Nonlinear LHTL employing a nonlinear inductor.

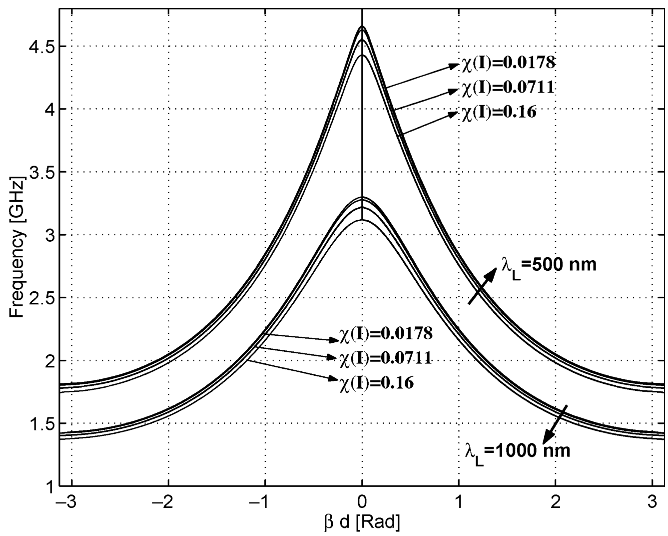

Fig. 10. Dispersion diagram of the LHTL as the shunt inductor varies.

$\mu\lambda \cot(t/\lambda)[1 + (4i^2/9i_c^2)]$ [14], which indicates that the kinetic inductance is a nonlinear function of the current (Fig. 9). Assume that the inductance per unit length of the CPW line as function of current is given by

$$L(I) = L_{external} + L_{kinetic}\left(1 + \chi(I^2)\right) \quad (2)$$

where χ is a phenomenological coefficient that models the effect of kinetic inductance nonlinearity. Fig. 10 depicts the dispersion diagram of the discussed LHTL for three different values of χ which corresponds to different values of DC bias current. Changing χ from 0.0178 to 0.16 provides a tuning range of 3.6% on the bragg frequency. This tuning can be specially useful when it is incorporated in the design of miniaturized resonators that uses a combination of LH and RH transmission lines [15]. This tuning capability helps to tune the resonator after its fabrication similar to the mechanism of the traditional tuning screws that are used to tune a filter.

V. Conclusion

A left-handed transmission line using superconductor material is studied. The effect of kinetic inductance on the dispersion diagram and frequency response of a left-handed transmission line is investigated. It is shown that employing CPW lines to realize LHTL can increase the effect of kinetic inductance by decreasing the spacing between the line and the ground plane. The effect of nonlinearity due to external DC bias current is studied. It is shown that external DC bias current can be employed in the LHTL structure to tune the index of refraction. Possible applications of a tunable SLHTL configuration are in the realization of miniaturized resonators and filters.

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